# Package 'SMMAL'

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Description Provides a pipeline for estimating the average treatment effect via semi-supervised learn
ing. Outcome regression is fit with cross-fitting using various machine learn-
ing method or user customized function. Doubly robust ATE estimation leverages both la-

ing method or user customized function. Doubly robust ATE estimation leverages both labeled and unlabeled data under a semi-supervised missing-data framework. For more details see Hou et al. (2021) <doi:10.48550/arxiv.2110.12336>. A detailed vignette is included.

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Title Semi-Supervised Estimation of Average Treatment Effects

## **Contents**

ate.SSL	2
cf	4
compute_parameter	6
cross_validation	7
param_fun	8
SMMAL	8
SMMAL ada lasso	Ç

ate.SSL

Index 12

```
ate. SSL Estimate Average Treatment Effect (ATE) via Semi-Supervised Learning
```

## Description

Estimate Average Treatment Effect (ATE) via Semi-Supervised Learning

## Usage

```
ate.SSL(
    Y,
    A,
    R,
    mu1,
    mu0,
    pi1,
    pi0,
    imp.A,
    imp.A1Y1,
    imp.A0Y1,
    min.pi = 0.05,
    max.pi = 0.95
```

## Arguments

Υ	Numeric vector. Observed outcomes for labeled data (with missing values for unlabelled).
Α	Numeric vector. Treatment indicator (1 for treated, 0 for control).
R	Logical or binary vector. Indicator for labeled data (1 if labeled, 0 if not).
mu1	Numeric vector. Estimated outcome regression $E[Y \mid A = 1, X]$ .
mu0	Numeric vector. Estimated outcome regression $E[Y \mid A = 0, X]$ .
pi1	Numeric vector. Estimated propensity scores $P(A = 1 \mid X)$ .
pi0	Numeric vector. Estimated propensity scores $P(A = 0 \mid X)$ .
imp.A	Numeric vector. Estimated treatment probabilities using surrogate covariates W.
imp.A1Y1	Numeric vector. Imputed $E[Y \mid A=1, W]$ using surrogate variables.
imp.A0Y1	Numeric vector. Imputed $E[Y\mid A=0,W]$ using surrogate variables.
min.pi	Numeric. Lower bound to truncate estimated propensity scores (default = $0.05$ ).
max.pi	Numeric. Upper bound to truncate estimated propensity scores (default = $0.95$ ).

ate.SSL 3

## **Details**

This function estimates the ATE in a semi-supervised setting, where outcomes are only observed for a subset of the sample. Surrogate variables and imputed models are used to leverage information from unlabelled data.

#### Value

A list containing:

est Estimated ATE.

se Estimated standard error of ATE.

## **Examples**

```
set.seed(123)
N <- 400
n <- 200 # Number of labeled observations
labeled_indices <- sample(1:N, n)</pre>
# Generate covariates and treatment
X <- rnorm(N)
A <- rbinom(N, 1, plogis(X))
# True potential outcomes
Y0_true <- X + rnorm(N)
Y1_{true} <- X + 1 + rnorm(N)
# Observed outcomes
Y_full <- ifelse(A == 1, Y1_true, Y0_true)
# Only labeled samples have observed Y
Y \leftarrow rep(NA, N)
Y[labeled_indices] <- Y_full[labeled_indices]
R <- rep(0, N); R[labeled_indices] <- 1</pre>
# Nuisance parameter estimates (can be replaced by actual model predictions)
mu1 < - X + 0.5
mu0 < - X - 0.5
pi1 <- plogis(X)
pi0 <- 1 - pi1
imp.A <- plogis(X)</pre>
imp.A1Y1 \leftarrow plogis(X) * (X + 0.5)
imp.A0Y1 \leftarrow (1 - plogis(X)) * (X - 0.5)
# Estimate ATE
result <- ate.SSL(
  Y = Y,
  A = A
  R = R,
  mu1 = mu1,
  mu0 = mu0,
  pi1 = pi1,
```

4 cf

```
pi0 = pi0,
imp.A = imp.A,
imp.A1Y1 = imp.A1Y1,
imp.A0Y1 = imp.A0Y1
)
print(result$est)
print(result$se)
```

cf

Cross-Fitting with Model Selection and Log Loss Evaluation

## **Description**

Trains and evaluates predictive models using cross-fitting across nfold folds, supporting multiple learner types. Outputs out-of-fold predictions and computes log\_loss for each hyperparameter tuning round to select the best-performing model.

## Usage

```
cf(
    Y,
    X,
    nfold,
    R,
    foldid,
    cf_model,
    sub_set = rep(TRUE, length(Y)),
    custom_model_fun = NULL
)
```

## **Arguments**

Υ	Numeric or factor vector. The response variable, either binary $(0/1)$ or continuous. Only labelled observations (where R = 1) are used.
Χ	Matrix or data frame. Predictor variables used for model training.
nfold	Integer. Number of cross-fitting folds.
R	Binary vector. Indicator of labelled data: 1 = labelled, 0 = unlabelled.
foldid	Integer vector. Fold assignments for cross-fitting (length equal to the full dataset).
cf_model	Character string. Specifies the model type. Must be one of " $xgboost$ ", " $bspline$ ", or "randomforest".
sub_set	Logical vector. Indicates which labelled samples to include in training.
custom_model_fu	ın

A logical or function. If NULL or FALSE, bypasses adaptive-LASSO feature selection. Otherwise, enables two-stage tuning inside compute\_parameter(). Defaults to all TRUE.

cf 5

#### **Details**

The function supports three learner types:

- **xgboost**: Gradient-boosted trees, tuning gamma across rounds.
- bspline: Logistic regression using B-spline basis expansions, tuning the number of knots.
- randomforest: Random forests, tuning nodesize.

Cross-fitting ensures that model evaluation is based on out-of-fold predictions, reducing overfitting. log\_loss is used as the evaluation metric to identify the best hyperparameter setting.

#### Value

A list containing:

models (Currently a placeholder) List of trained models per fold and tuning round.

**predictions** List of out-of-fold predictions for each of the 5 tuning rounds.

log\_losses Numeric vector of log loss values for each tuning round.

**best\_rounds\_index** Integer index (1–5) of the round achieving the lowest log\_loss.

best\_rounds\_log\_losses Minimum log\_loss value achieved across rounds.

best\_rounds\_prediction Vector of out-of-fold predictions from the best tuning round.

### **Examples**

```
set.seed(123)
X \leftarrow matrix(rnorm(N * 5), nrow = N, ncol = 5)
# Simulate treatment assignment
A \leftarrow rbinom(N, 1, plogis(X[, 1] - 0.5 * X[, 2]))
# Simulate outcome
Y_{full} \leftarrow rbinom(N, 1, plogis(0.5 * X[, 1] - 0.25 * X[, 3]))
# Introduce some missingness to simulate semi-supervised data
Y <- Y_full
Y[sample(1:N, size = N/4)] <- NA # 25% missing
# Create R vector (labelled = 1, unlabelled = 0)
R <- ifelse(!is.na(Y), 1, 0)</pre>
# Cross-validation fold assignment
foldid <- sample(rep(1:5, length.out = N))</pre>
# Run cf with glm model
result <- cf(Y = Y, X = X, nfold = 5, R = R, foldid = foldid, cf_model = "glm")
# Examine output
print(result$log_losses)
print(result$best_rounds_index)
```

6 compute\_parameter

compute\_parameter

Estimate Nuisance Parameters for Semi-Supervised ATE Estimation

#### **Description**

Computes nuisance functions including conditional expectations and propensity scores using cross-fitting, separately for labelled and unlabelled data. These estimates are essential inputs for doubly robust or semi-supervised average treatment effect (ATE) estimators.

#### Usage

```
compute_parameter(nfold, Y, A, X, S, W, foldid, R, cf_model, custom_model_fun)
```

#### **Arguments**

nfold	Integer. Number of cross-fitting folds.
Υ	Numeric vector. Outcome variable. Can contain NAs for unlabelled observations.
Α	Numeric vector. Treatment assignment indicator (0 or 1). Can contain NAs.
Χ	Matrix or data frame. Covariates used for outcome and propensity score models.
S	Matrix or data frame. Additional covariates used only in imputation models.
W	Matrix or data frame. Combined set of covariates (typically cbind(X, S)).
foldid	Integer vector. Fold assignments for cross-fitting.
R	Binary vector. Label indicator: 1 = labelled (observed A and Y), 0 = unlabelled.
cf_model	Function. A user-supplied cross-fitting wrapper function (e.g., based on Super Learner or other learners).
custom_model_f	un
	A logical or function. If NULL or FALSE, bypasses adaptive-LASSO feature se-

## **Details**

This function applies cross-fitting to estimate all required nuisance functions for semi-supervised or doubly robust ATE estimators. Separate models are fit for the labelled dataset and the full dataset (for imputation).

lection. Otherwise, enables two-stage tuning inside compute\_parameter().

#### Value

A named list of estimated nuisance parameters (each a numeric vector):

```
pi1.bs Estimated propensity score P(A=1\mid X).

pi0.bs Estimated propensity score P(A=0\mid X) (computed as 1 - pi1.bs).

mu1.bs Estimated outcome regression E[Y\mid A=1,X].

mu0.bs Estimated outcome regression E[Y\mid A=0,X].

cap_pi1.bs Estimated imputed propensity score P(A=1\mid W).
```

cross\_validation 7

```
cap_pi0.bs Estimated imputed propensity score P(A=0 \mid W) (computed as 1 - cap_pi1.bs). 
m1.bs Estimated imputed outcome regression E[Y \mid A=1, W]. 
m0.bs Estimated imputed outcome regression E[Y \mid A=0, W].
```

#### See Also

cf

cross\_validation

Assign Cross-Validation Folds for Labelled and Unlabelled Data

## **Description**

Creates fold assignments for both labelled and unlabelled data using stratified random sampling, ensuring an approximately equal number of samples per fold within each group.

### Usage

```
cross_validation(N, nfold, A, Y)
```

## **Arguments**

N	Integer. Total number of observations in the dataset.
nfold	Integer. Number of folds to assign for cross-validation.
A	Numeric vector. Treatment assignment indicator (may contain NA for unlabelled samples).
Υ	Numeric vector. Outcome variable (may contain NA for unlabelled samples).

## **Details**

The function first separates observations into labelled and unlabelled groups based on the availability of both treatment (A) and outcome (Y). Within each group, fold assignments are randomly assigned to ensure approximately balanced sample sizes across folds. This setup supports semi-supervised learning workflows by maintaining structure between labelled and unlabelled data during cross-fitting.

#### Value

A list containing:

**R** Binary vector of length N, where 1 indicates labelled observations (non-missing A and Y), and 0 indicates unlabelled observations.

foldid Integer vector of length N. Fold assignments (from 1 to nfold) for use in cross-validation.

8 SMMAL

### **Examples**

```
set.seed(123) \\ N <- 100 \\ A <- sample(c(0, 1, NA), size = N, replace = TRUE, prob = c(0.45, 0.45, 0.10)) \\ Y <- sample(c(0, 1, NA), size = N, replace = TRUE, prob = c(0.45, 0.45, 0.10)) \\ \# Assign 5 folds for cross-fitting \\ result <- cross_validation(N = N, nfold = 5, A = A, Y = Y) \\ table(result\$R) \# Check number of labelled vs unlabelled \\ table(result\$foldid) \# Check how folds are distributed
```

param\_fun

Parameter grid function

## Description

Returns two list of hyperparameters for model tuning.

#### Usage

```
param_fun()
```

## Value

```
A list, e.g., list(ridge = ..., lambda = ...)
```

SMMAL

Estimate Average Treatment Effect (ATE) via Semi-Supervised Learning Pipeline

#### **Description**

Executes a full semi-supervised ATE estimation pipeline. This includes cross-validation fold assignment, feature selection via adaptive LASSO, model fitting using a specified learner (e.g., bspline, xgboost, or random forest), and doubly robust ATE estimation that leverages both labelled and unlabelled data.

## Usage

```
SMMAL(Y, A, S, X, nfold = 5, cf_model = "bspline", custom_model_fun = NULL)
```

SMMAL\_ada\_lasso 9

#### **Arguments**

Υ	$Numeric\ vector.\ Outcome\ variable\ (may\ contain\ NA\ for\ unlabelled\ observations).$	
A	Numeric vector. Treatment indicator (1 = treated, $0 = \text{control}$ ). May contain NA for unlabelled observations.	
S	Matrix or data frame. Surrogate variables used only in imputation models.	
X	Matrix or data frame. Main covariates used for outcome and propensity score modeling.	
nfold	Integer. Number of cross-validation folds. Default is 5.	
cf_model	Character string. Modeling method to use in cross-fitting. One of "bspline", "xgboost", or "randomforest". Default is "bspline".	
custom model fu	ın	

A logical or function. If NULL or FALSE, bypasses adaptive-LASSO feature selection. Otherwise, enables two-stage tuning inside compute\_parameter().

#### **Details**

The pipeline first selects important covariates via adaptive LASSO. Then, it fits nuisance functions (outcome regressions and propensity scores) using cross-fitting with the specified learner. Finally, it applies a doubly robust estimator that integrates information from both labelled and unlabelled observations to estimate the ATE.

## Value

A list containing:

est Estimated Average Treatment Effect (ATE).

se Estimated standard error of the ATE.

#### See Also

```
cf, compute_parameter, cross_validation, ate.SSL
```

SMMAL_ada_lasso	Adaptive LASSO with Cross-Validation	

## **Description**

Performs adaptive LASSO for binary outcomes by first fitting a ridge regression to compute penalty factors, and then running cross-validated lasso fits over a grid of lambda values.

10 SMMAL\_ada\_lasso

#### Usage

```
SMMAL_ada_lasso(
   X,
   Y,
   X_full,
   foldid,
   foldid_labelled,
   sub_set,
   labeled_indices,
   nfold,
   log_loss
)
```

## **Arguments**

X A numeric matrix of predictors (n observations  $\times$  p features).

Y A numeric or integer vector of binary outcomes (length n).

X\_full The full matrix of predictors for all observations.

foldid A vector assigning each observation (labelled or unlabelled) to a fold.

foldid\_labelled

An integer vector (length n) of fold assignments for labeled observations. Values should run from 1 to nfold; other values (e.g., NA) indicate unlabeled or held-out

rows.

sub\_set A logical or integer vector indicating which rows of X/Y are used in supervised

CV.

labeled\_indices

An integer or logical vector indicating which rows have non-missing outcomes.

nfold A single integer specifying the number of CV folds (e.g., 5 or 10).

log\_loss A function of the form function(true\_labels, pred\_probs) that returns a

single log-loss numeric.

#### **Details**

This function expects that a parameter-generating function param\_fun() is available in the package, returning a list with elements \$ridge (a vector of ridge penalty values) and \$lambda (a vector of lasso penalty values). Internally, it:

- 1. Fits a ridge-penalized logistic regression on all data to obtain coefficients.
- 2. Computes penalty factors as 1 / (abs(coef) + 1e-4).
- 3. For each ridge value, runs n-fold CV over lambda values with glmnet(..., alpha=1).
- 4. Records predictions on held-out folds, computes log-loss for each lambda, and selects the lambda with minimum log-loss.
- 5. Returns a list of CV-predicted probability vectors (one vector per ridge value).

SMMAL\_ada\_lasso 11

#### Value

A list of length equal to the number of ridge penalty values provided by param\_fun(). Each element is a numeric vector (length = n) containing cross-validated predicted probabilities for the best lambda under that ridge penalty.

#### **Examples**

```
# Assume param_fun() is defined elsewhere and returns:
# list(ridge = c(0.01, 0.1, 1), lambda = exp(seq(log(0.001), log(1), length = 50)))
# Simulate small data:
set.seed(123)
n <- 100; p <- 10
X <- matrix(rnorm(n * p), nrow = n)</pre>
true_beta <- c(rep(1.5, 3), rep(0, p - 3))
lin <- X %*% true_beta</pre>
probs <- 1 / (1 + \exp(-1in))
Y <- rbinom(n, 1, probs)
# Create fold assignments for labeled observations:
labeled <- sample(c(TRUE, FALSE), n, replace = TRUE, prob = c(0.8, 0.2))
foldid_labelled <- rep(NA_integer_, n)</pre>
foldid_labelled[labeled] <- sample(1:5, sum(labeled), replace = TRUE)</pre>
sub set
                <- labeled
labeled_indices <- which(labeled)</pre>
# For simplicity, assign foldid to all observations (labeled & unlabeled)
foldid <- sample(1:5, n, replace = TRUE)</pre>
# Define a simple log-loss function:
log_loss_fn <- function(true, pred) {</pre>
  eps <- 1e-15
  pred_clipped <- pmin(pmax(pred, eps), 1 - eps)</pre>
  -mean(true * log(pred_clipped) + (1 - true) * log(1 - pred_clipped))
}
# Call SMMAL_ada_lasso with all required args:
results <- SMMAL_ada_lasso(
  X = X
  Y = Y,
  X_full = X, # Here full data same as X for example
  foldid = foldid,
  foldid_labelled = foldid_labelled,
  sub_set = sub_set,
  labeled_indices = labeled_indices,
  nfold = 5,
  log_loss = log_loss_fn
```

# 'results' is a list (one element per ridge value), each a numeric vector of CV predictions.

## **Index**

```
ate.SSL, 2, 9

cf, 4, 7, 9

compute_parameter, 6, 9

cross_validation, 7, 9

param_fun, 8

SMMAL, 8

SMMAL_ada_lasso, 9
```