Package 'syt'

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Type Package

Title Young Tableaux

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Description Deals with Young tableaux (field of combinatorics). For standard Young tabeaux, performs enumeration, counting, random generation, the Robinson-Schensted correspondence, and conversion to and from paths on the Young lattice. Also performs enumeration and counting of semistandard Young tableaux, enumeration of skew semistandard Young tableaux, and computation of Kostka numbers.

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URL https://github.com/stla/syt

BugReports https://github.com/stla/syt/issues

Imports Matrix, partitions, utils

Suggests testthat

Encoding UTF-8

RoxygenNote 7.3.1

NeedsCompilation no

Repository CRAN

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R topics documented:

`all_ssSkewTableaux` *Semistandard skew tableaux*

Description

Enumeration of all semistandard skew tableaux with given shape and given maximum entry.

Usage

```
all_ssSkewTableaux(lambda, mu, n)
```

Arguments

`lambda, mu`	integer partitions defining the skew partition: `lambda` is the outer partition and `mu` is the inner partition (so `mu` must be a subpartition of `lambda`)
`n`	a positive integer, the maximum entry of the skew tableaux

Value

The list of all semistandard skew tableaux whose shape is the skew partition defined by `lambda` and `mu` and with maximum entry `n`.

Examples

```
ssstx <- all_ssSkewTableaux(c(4, 3, 1), c(2, 2), 2)
lapply(ssstx, prettySkewTableau)
```

`all_ssytx` *Enumeration of semistandard Young tableaux*

Description

Generates all semistandard Young tableaux of a given shape and filled with integers between `1` and a given `n`.

Usage

```
all_ssytx(lambda, n)
```

Arguments

`lambda`	an integer partition, the shape
`n`	an integer, the maximum value of the entries (the minimum value is `1`)

Value

List of all semistandard Young tableaux with shape `lambda` and filled with integers between `1` and `n`.

Examples

```
ssytx <- all_ssytx(c(2, 1), 3)
lapply(ssytx, prettyTableau)
```

all_sytx *Enumeration of standard Young tableaux*

Description

Generates all standard Young tableaux of a given shape.

Usage

```
all_sytx(lambda)
```

Arguments

lambda	shape, an integer partition

Value

A list of standard Young tableaux.

Examples

```
sytx <- all_sytx(c(5, 2))
lapply(sytx, prettyTableau)
```

ballot2syt *Tableau as ballot sequence*

Description

Converts a ballot sequence to its corresponding standard Young tableau.

Usage

```
ballot2syt(a)
```

Arguments

a	ballot sequence

Value

A standard Young tableau.

See Also

syt2ballot

Examples

```
a <- c(1,1,2,3,2,1)
ballot2syt(a)
```

---

count_ssytx *Number of semistandard Young tableaux*

Description

Number of semistandard Young tableaux of a given shape and filled with integers between 1 and a given n.

Usage

```
count_ssytx(lambda, n)
```

Arguments

lambda	an integer partition, the shape
n	an integer, the maximum value of the entries (the minimum value is 1)

Value

The number of semistandard Young tableaux with shape lambda and filled with integers between 1 and n.

Examples

```
count_ssytx(c(4, 3, 3, 2), 5)
```

---

count_sytx *Number of standard Young tableaux*

Description

Number of standard Young tableaux of a given shape.

Usage

```
count_sytx(lambda)
```

Arguments

lambda	an integer partition, the shape

Value

An integer, the number of standard Young tableaux of shape `lambda`.

See Also

`all_sytx`

Examples

```
count_sytx(c(5,4,1))
length(all_sytx(c(5,4,1)))
```

`dualSkewTableau` *Dual skew tableau*

Description

Returns the dual (skew) tableau of a skew tableau.

Usage

```
dualSkewTableau(skewTableau)
```

Arguments

`skewTableau`	a skew tableau

Value

A skew tableau.

Examples

```
tbl <- list(c(NA, NA, 1, 1), c(NA, 1), c(1, 2))
dtbl <- dualSkewTableau(tbl)
prettySkewTableau(dtbl)
```

dualsyt *Dual tableau*

Description

The dual standard Young tableau of a standard Young tableau.

Usage

```
dualsyt(syt)
```

Arguments

syt	standard Young tableau

Value

A standard Young tableau.

Examples

```
syt <- list(c(1,2,6), c(3,5), 4)
dualsyt(syt)
```

dualTableau *Dual tableau*

Description

The dual tableau of a tableau (mirror image to the main diagonal).

Usage

```
dualTableau(tableau)
```

Arguments

tableau	a tableau

Value

A tableau.

Examples

```
tbl <- list(c("a", "s", "e", "f"), c("f", "o"), c("u"))
dualTableau(tbl)
```

firstsyt *First tableau of a given shape*

Description

Returns the "first" standard Young tableau of a given shape.

Usage

```
firstsyt(lambda)
```

Arguments

lambda	the shape, an integer partition

Value

A standard Young tableau.

Examples

```
firstsyt(c(4,2,1))
```

gprocess2syt *Growth process to tableau*

Description

Converts a growth process of integer partitions to its corresponding standard Young tableau.

Usage

```
gprocess2syt(path)
```

Arguments

path	a path of the Young graph from the root vertex, given as a list of integer partitions

Value

A standard Young tableau.

See Also

syt2gprocess

Examples

```
path <- list(1, 2, c(2,1), c(3,1), c(3,1,1))
gprocess2syt(path)
```

`hooklengths` *Hook lengths*

Description

Hook lengths of a given integer partition.

Usage

```
hooklengths(lambda)
```

Arguments

`lambda`	an integer partition

Value

The hook lengths of the partition, given in a list.

See Also

`hooks`

Examples

```
hooklengths(c(4,2))
```

`hooks` *Hooks*

Description

Hooks of a given integer partition.

Usage

```
hooks(lambda)
```

Arguments

`lambda`	integer partition

Value

The hooks of the partition in a list.

See Also

hooklengths

Examples

```
hooks(c(4,2))
```

`isSemistandardSkewTableau` *Check whether a skew tableau is semistandard*

Description

Check whether a skew tableau is a semistandard skew tableau.

Usage

```
isSemistandardSkewTableau(skewTableau)
```

Arguments

skewTableau	a skew tableau

Value

A Boolean value.

Examples

```
tbl <- list(c(NA, NA, 1, 1), c(NA, 1), c(1, 2))
isSemistandardSkewTableau(tbl)
```

isSkewTableau *Check whether a tableau is a skew tableau*

Description

Check whether a tableau is a skew tableau.

Usage

```
isSkewTableau(tableau)
```

Arguments

tableau	a tableau

Value

A Boolean value.

Examples

```
tbl <- list(c(NA, NA, 1, 1), c(NA, 1), c(1, 2))
isSkewTableau(tbl)
```

isSSYT *Checks whether a tableau is semistandard*

Description

Checks whether a tableau is a semistandard Young tableau.

Usage

```
isSSYT(tableau)
```

Arguments

tableau	a tableau

Value

A Boolean value.

Examples

```
tbl <- list(c(1,2,6), c(5,5), 7)
isSSYT(tbl)
```

isStandardSkewTableau *Check whether a skew tableau is standard*

Description

Check whether a skew tableau is a standard skew tableau.

Usage

```
isStandardSkewTableau(skewTableau)
```

Arguments

skewTableau	a skew tableau

Value

A Boolean value.

Examples

```
tbl <- list(c(NA, NA, 1, 1), c(NA, 1), c(1, 2))
isStandardSkewTableau(tbl)
```

isSYT *Checks whether a tableau is standard*

Description

Checks whether a tableau is a standard Young tableau.

Usage

```
isSYT(tableau)
```

Arguments

tableau	a tableau

Value

A Boolean value.

Examples

```
tbl <- list(c(1,2,6), c(3,5), 4)
isSYT(tbl)
```

KostkaNumber *Kostka numbers*

Description

Computes a Kostka number.

Usage

```
KostkaNumber(lambda, mu)
```

Arguments

lambda	an integer partition
mu	an integer vector whose sum equals the weight (i.e. the sum) of lambda

Details

The Kostka number $K(\lambda, \mu)$ is the number of semistandard Young tableaux with shape λ and weight μ. It does not depend on the order of the elements of μ (so one can always take an integer partition for μ). The *weight* is the vector whose i-th element is the number of occurences of i in the tableau.

Value

The Kostka number corresponding to lambda and mu.

Examples

```
KostkaNumber(c(3,2), c(1,1,1,2))
KostkaNumber(c(3,2), c(1,1,2,1))
KostkaNumber(c(3,2), c(1,2,1,1))
KostkaNumber(c(3,2), c(2,1,1,1))
```

matrix2syt *Standard Young tableau from a matrix*

Description

Converts a matrix to a standard Young tableau.

Usage

```
matrix2syt(M)
```

Arguments

M	a matrix

Value

A standard Young tableau.

See Also

syt2matrix

Examples

```
M <- rbind(c(1,2,6), c(3,5,0), c(4,0,0))
matrix2syt(M)
```

---

nextsyt *Next tableau*

Description

Given a standard Young tableau, returns the "next" one having the same shape.

Usage

```
nextsyt(syt)
```

Arguments

syt	a standard Young tableau

Value

A standard Young tableau of the same shape as `syt`, or `NULL` if `syt` is the last standard Young tableau of this shape.

Examples

```
syt <- firstsyt(c(4,2,1))
nextsyt(syt)
```

prettySkewTableau *Pretty skew tableau*

Description

Pretty form of a skew tableau.

Usage

```
prettySkewTableau(skewTableau)
```

Arguments

skewTableau	a skew tableau

Value

A 'noquote' character matrix.

Examples

```
tbl <- list(c(NA, NA, 1, 1), c(NA, 1), c(1, 2))
prettySkewTableau(tbl)
```

prettyTableau *Pretty tableau*

Description

Pretty form of a tableau.

Usage

```
prettyTableau(tableau)
```

Arguments

tableau	a tableau

Value

A 'noquote' character matrix.

Examples

```
tbl <- list(c(0, 2, 1, 1), c(4, 1), c(1, 2))
prettyTableau(tbl)
```

rgprocess *Plancherel growth process*

Description

Samples a path of the Young graph according to the Plancherel growth process.

Usage

```
rgprocess(n)
```

Arguments

n	the size of the path to be sampled

Value

The path as a list, starting from the root vertex 1.

See Also

gprocess2syt and syt2gprocess to convert a Young path to a standard Young tableau and conversely.

Examples

```
rgprocess(7)
```

RS *Robinson-Schensted correspondence*

Description

Pair of standard Young tableaux given from a permutation by the Robinson-Schensted correspondence.

Usage

```
RS(sigma)
```

Arguments

sigma	a permutation given as a vector of integers

Value

A list of two standard Young tableaux.

Examples

```
RS(c(1, 3, 6, 4, 7, 5, 2))
```

---

rsyt *Random standard Young tableau*

Description

Uniform sampling of a standard Young tableau of a given shape.

Usage

```
rsyt(lambda)
```

Arguments

lambda	shape, an integer partition

Value

A standard Young tableau of shape `lambda`.

Examples

```
rsyt(c(7,3,1))
```

---

syt2ballot *Tableau as ballot sequence*

Description

Converts a standard Young tableau to its corresponding ballot sequence.

Usage

```
syt2ballot(syt)
```

Arguments

syt	standard Young tableau

Value

A ballot sequence.

See Also

ballot2syt

Examples

```
syt <- list(c(1,2,6), c(3,5), 4)
syt2ballot(syt)
```

`syt2gprocess` *Tableau as growth process*

Description

Converts a standard Young tableau to its corresponding growth process of partitions.

Usage

```
syt2gprocess(syt)
```

Arguments

`syt`	standard Young tableau

Value

A list of integer partitions, representing a path of the Young graph starting from the root vertex.

See Also

gprocess2syt

Examples

```
syt <- list(c(1,2,4), 3, 5)
syt2gprocess(syt)
```

syt2matrix *Standard Young tableau as sparse matrix*

Description

Representation of a standard Young tableau as a sparse matrix.

Usage

```
syt2matrix(syt)
```

Arguments

syt	a standard Young tableau

Value

A sparse matrix.

Note

This function is the same as `tableau2matrix` except that in addition it checks that the given tableau is a standard Young tableau.

See Also

`matrix2syt`

Examples

```
syt <- list(c(1,2,6), c(3,5), 4)
syt2matrix(syt)
```

tableau2matrix *Tableau as sparse matrix*

Description

Representation of a tableau as a sparse matrix; only for a tableau with numeric or logical entries.

Usage

```
tableau2matrix(tableau)
```

Arguments

tableau	a tableau with numeric or logical entries

Value

A sparse matrix.

Examples

```
syt <- list(c(1,2,6), c(3,5), 4)
syt2matrix(syt)
```

`tableauShape` *Shape of a tableau*

Description

The shape of a tableau.

Usage

```
tableauShape(tableau)
```

Arguments

`tableau`	a tableau (list of vectors having the same mode)

Value

The shape of the tableau (an integer partition).

Examples

```
tableau <- list(c(2, 1, 3), c(5, 2))
tableauShape(tableau)
```

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